EXPLORING PERDE-SPACE: 3-D TONNETZ VISUALIZATION FOR MAKAM MUSIC

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ABSTRACT

This paper introduces a similarity and weight metric designed to assess the distance between pitches and melodic fragments within the framework of Turkish Makam Music. Our approach hinges on adapting the Riemannian Tonnetz to represent pitch-based elements, termed "perde" (pitch), or "çeşni" (tetrachordal material for melodic fragments), within a three-dimensional vector space tailored specifically for Turkish Makam Music. We demonstrate the efficacy of this space as a reliable approximation for gauging similarity between musical objects and estimating the cost associated with traversing the music network using different distance metrics namely Euclidean, Minkowski, Manhattan and Cosine, thereby facilitating the construction of a weighted version thereof. Taking weighted relationships into consideration, this metric enables the computation of the cost associated with potential paths between musical objects and furnishes a new predictive tool for supervised learning models. Following the exposition of the geometric model and calculation methodology, we undertake a comparative analysis with results derived from a sample annotated by expert practitioners, revealing a notably high degree of correlation between our algorithm utilizing cosine distance and expert ratings, Spearman's rho = 0.828 with p-value < .001.

1. MOTIVATION AND RESEARCH

This paper aims to investigate the feasibility of mapping the pitch and melodic components, namely "*perde*" and "*çeşni*," within Turkish Makam Music onto a geometric or topological framework. This framework could serve as a basis for quantifying distance and similarity among these elements. The impetus for this inquiry emerged from Project makamNetz, which endeavors to depict all structures of "*makam*" and *"çeşni*," along with their interrelations, as an extensive network or map. Upon scrutinizing the topological configurations of this network, it became apparent that the relationships themselves do not possess uniform significance. This concern became particularly evident when delineating paths between different nodes, as the network assigned equal weight to paths of equivalent total distance, despite their varying frequencies in practical application.

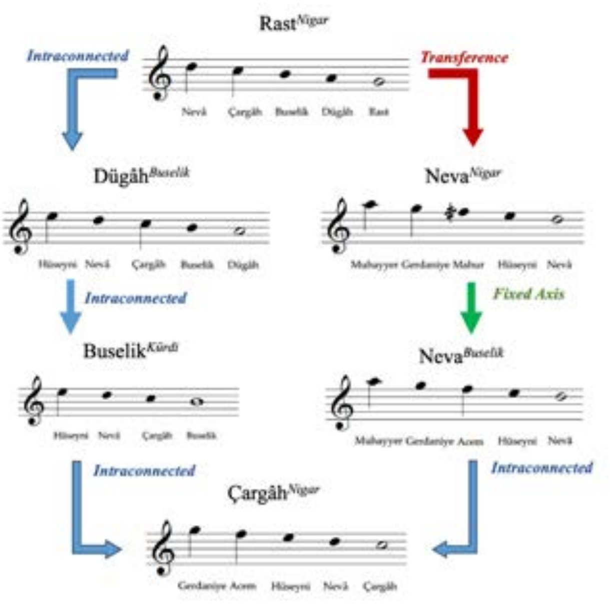

Figure 1. Two melodic progressions from Turkish makam music.

The makam space can be understood as comprising different *çeşni* elements, referred to as CMAKAM. Each element in this space consists of 3 to 5 pitches or "*perde*." These melodic structures evolve according to specific relational principles. Figure 1 depicts two configurations of Perde^{Çeşni} progressions outlining the structural body of two melodic lines . while specifying the *perde* collection from which various melodic structures can emerge. Both melodic lines begin and end on the same spaces, commencing the melodic progression with Rast^{Nigar} and concluding with Çargah^{Nigar}. However, the paths they take to reach their destination differ. The first progression (intraconnected) illustrates a smoother transition achieved by adding new notes to the existing melodic formula or by shifting the tonic of the tetrachordal or pentachordal structure. The second progression, though more unconventional, is still feasible. It involves transferring pitches entirely to a new axis—a fifth or fourth related to the original sequence (transference, indicated by red arrows)—or simply altering some scale degrees on the same axis (fixed axis, indicated by green arrows). From a progressional standpoint, relationships involving common tones or direct transpositions are considered natural and are frequently encountered within various makams.

However, progressions utilizing varying *perde* material that share the same axis—which represent modal shifts—have not been previously modeled within Turkish Makam Music context and present a significant challenge. Without a proper hierarchical measurement principle, the two progressions depicted below exhibit similar topology and are considered equivalent in terms of distance, which is inaccurate from both practical and cognitive perspectives. Our aim is to model this differentiation, thereby creating a weighted network where different progressions carry distinct weights, reflecting varying path distances.

2. DISTANCE IN MUSICAL SPACE

The question of distance has garnered widespread recognition and exploration in music theory, with various spatial representations offered for different musical styles, particularly within the realm of common practice tonality. While some theorists grounded their assertions in music-theoretical principles, others rooted their work in cognitive approaches. Though this paper does not aim to provide an exhaustive survey of these approaches, it is pertinent to reference those that informed this study. [1] presents a hierarchical model of pitch relationships within a tonal context, which is supported by the cognitive research of Deutsch and Feroe [2]. This model investigates pitch distance across five distinct levels: octave, fifth, third, diatonic, and chromatic. Drawing from similar conceptual foundations, Lerdahl establishes a chordal space, wherein chords are interconnected through alternation PL transformations along the horizontal axis and major thirds along the vertical axis. In these representations, octaves, fifths, and thirds are accorded priority as the primary constituents of both pitch and chordal spaces.

Alternative approaches to measuring distance have also been proposed by Tymoczko, [3] in his seminal work "A Geometry of Music," where he delved into alternative pathways between scales, chordal objects, and voice-leading schemas. Tymocko's work predominantly relies on geometric representations of musical entities as nodes and links, or some variation of vector space. Both Tymocko and Lerdahl contend that distance in music constitutes a cognitive phenomenon and can be geometrically modeled, allowing for specific calculations. The inquiry into distance extends beyond the confines of music theory. A recent study employing network analysis of 456 annotated jazz improvisation transcriptions, coupled with cognitive experiments, revealed that perceived distance, as a measure of similarity, aligns with network distance measures (Merseal et al., [4]).

3. MAKAM PERDE SPACE

Prioritizing fifths and thirds is not unique to Western tonal context. A similar framework is applicable in Turkish Makam Music when deriving "*perde*" relationships, thus constructing a "Makam Perde" space or PMAKAM [1]. Figure 2 [2] depicts a section of the makam/perde space as a 2D Tonnetz representation. The horizontal dimension illustrates fifth, while verticality denotes third relations. In Turkish makam music, thirds and fifths are tuned according to the principles of the harmonic series. However, attempting to construct Turkish *perde* space based on these principles results in an impractically large conceptual Tonnetz where many closely related "*perde*" are distantly spaced. This is because constructing the vertical dimension based on natural thirds would yield "*perde*" tunings that are not practically used. Consequently, the result is a large horizontal Tonnetz that cannot form a cycle.

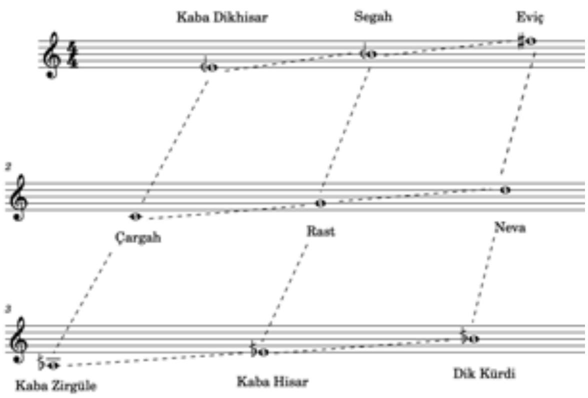

Figure 2. Section of makam perde space in 2d Tonnetz.

To address this question, we constructed a tonnetz without assuming octave equivalence, thereby contributing to the generation of "perde" space. The resulting geometry is depicted in Figure 3. According to this figure, Turkish Makam *Perde* Space (PMAKAM) is conceptualized as five fundamental layers interconnected via octave relations, as indicated by vertical slashed lines. It's worth noting that the upper-left and lower-right layers are also connected via octave relations, assuming a toroidal shape. Essentially, this space indicates the distance between makam objects in terms of thirds, fifths, or octaves. For example, on the third layer in the middle, we observe two frequently used "*perde*" called "*Rast*" and "*Neva*." The former is a fifth above the latter, positioned one step to the right on the x-axis. Similarly, "*Segâh*" is a natural third above "*Rast*," so it is one step up on the y-axis. "*Gerdaniye*," connected to "*Rast*" via a slashed line, is an octave above, shifting the layer one step up on the z-axis, thus serving as special connectors between layers. It's important to note that the tonnetz representation does not refer to static frequencies but rather denotes third, octave, and fifth relations, historically the building blocks of Turkish Makam Space.

[1] When naming PMAKAM and CMAKAM we follow similar naming convention to Hook [5], an important inspiration in formulating the spatial and topological properties of the space.

[2] This representation is presented in TUMAC BSE No.25: "Multi-dimensionality in Makam Musics" by Dr. Michael Ellison https://www.youtube.com/watch?v=6-SSscA8ZRc

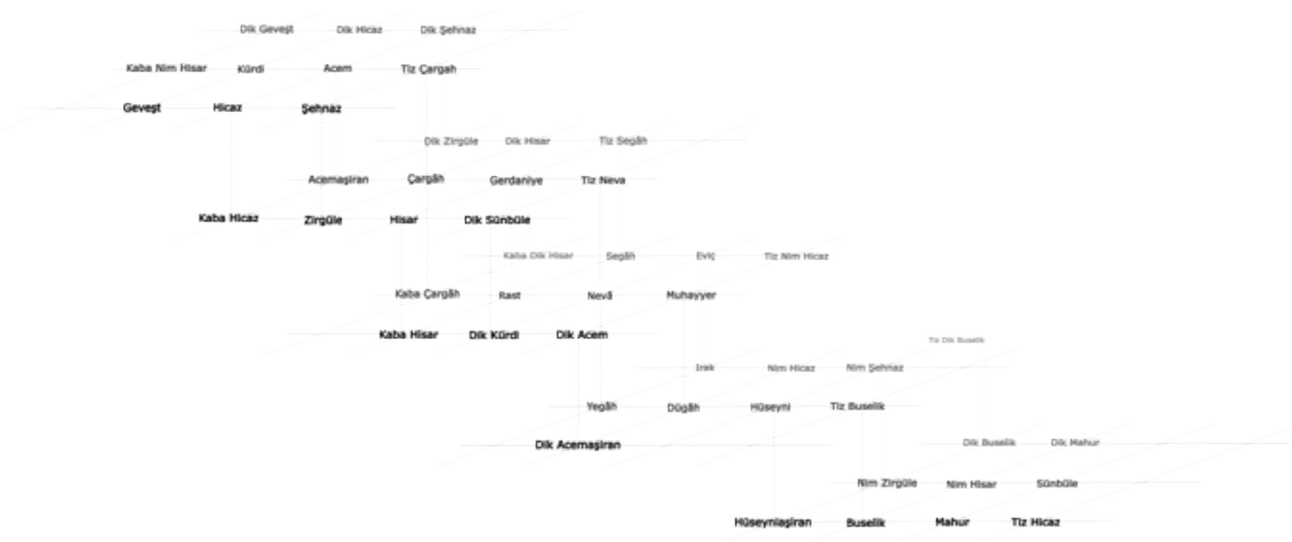

Figure 3. Three Dimensional Makam Tonnetz.

4. MODELING AND ALGORITHM

Related to our first research question we discovered that the 3D tonnetz model of PMAKAM can be used to calculate distance between different cesnis particularly the ones that has primary axis relation. In this model a perde is a vector in R^3 and a cesni is 3 by n matrix composed of 3 to 5 perdes. In this space we utilized different distance calculation methods and compared the results with the survey results we obtained from 16 expert raters. We have used 3 different algorithms to calculate the distance between 2 sequences and used the average of these 3 algorithms as the final distance measurement between these 2 sequences. In the 1st algorithm we measured the distance between each perde in both sequences and these distances are cumulated to give the net distance. If one of the sequences has fewer perdes compared to the other one, then the missing perdes are substituted by the starting perde. Formally we can define the process as below:

$$P_{axis} = [0,0,0]$$
$$PTonnetz = \{perde \mid perde = [x,y,z]^T - P_{axis}^T \quad , x,y,z \in \mathbb{Z}\} \qquad (1)$$

Thus we can state that $PTonnetz$ is a geometric representation of a conceptual perde space whose structure depends on primary axis perde situated on P_{axis}. The resulting space is a subspace of $\mathbb{Z}^3$. Next we define a two cesni sequence to compare:

$$Cesni_1 \in PTonnetz^{1xn} \text{ and } Cesni_2 \in PTonnetz^{1xm}$$
$$CostFunction = \{Euclidean, Minkovski, Manhattan, Cosine\}$$
$$Distance(C_1, C_2) = \sum_{i=1}^{k} CostFunction(x_i, y_i)$$
$$\text{where } x_i \in C_1, y_i \in C_2 \text{ and } k = max(m,n) \qquad (2)$$

We can now define the algorithms to calculate distance. The algorithm 1 uses the following formula:

```
Step 1. if n > m:
     extraindex = {i for m < i <= n}
     for j in extraindex:
          Cesni_1[j] = {P_axis}
Step 2. elif
     extraindex = {i for n < i <= m}
     for j in extraindex:
          Cesni_2[j] = [P_axis]

Step 3. distance = Distance(Cesni_1,
     &Cesni_2)
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The missing perdes are substituted by a randomly selected perde within the sequence in the 2nd algorithm. Thus we simply replace the code in step 1 and 2 that assigns the extra index of Cesnis to P_{axis} with random element from $PTonnetz$.

The 3rd algorithm calculates all pairwise distances within the sequence and cumulates them. The distances in the 3-dimensional space are calculated with 4 different distance calculations: Euclidean, Manhattan, Cosine and Minkowski (p = 10) methods. In the 3rd algorithm, the minimum of these 4 distance measurements is taken into account. Since it is simpler to show mathematically we can formalize it as below:

$$Distance(C_1, C_2) =$$
$$min(\sum_{i=1}^{n}\sum_{j=1}^{m} CostFunction(x_i, y_j)), x_i \in C_1, y_j \in C_2 \qquad (3)$$

We also have to note that in all algorithms after calculating all pairs in question we scale the cost function to a value between 1 and 5 matching maximum cost to 5 and minimum to 1. This helps us to compare and visualize the results with the experiment we obtained from the participants and formalize the third algorithm. [3] .

[3] Detailed computations can be accessed via GitHub (https://github.com/yavuzburuk/makamnetz)

Figure 4. Experts' Evaluations versus Distance Functions.

5. EXPERIMENT RESULTS WITH EXPERTS

The experiment engaged sixteen expert raters, all possessing formal backgrounds in Turkish music conservatory education and aged between 25 and 45 years old. The raters represented a diverse group including university faculty members, Ph.D. and M.A. students, as well as professional practitioners. Each rater was presented with a list of 36 Perde^{Çeşni} changes, encompassing fixed-axis transformations within Turkish music theory, such as transitions from Dügah^{Hicaz} to Dügah^{Uşşak} or Dügah^{Saba} to Dügah^{Buselik} … etc. Their task involved rating the perceived difficulty of executing each Perde^{Çeşni} change on a scale ranging from 1 to 5. Ratings of 1 indicated the smoothest and easiest transitions, while ratings of 5 indicated the most demanding and challenging transitions.

Subsequently, the ratings provided by the expert raters underwent inter-rater reliability analysis using Krippendorff's Alpha. The resulting Krippendorff's Alpha value, K-Alpha = 0.3846, suggested a moderate level of agreement among the raters. This indicated that while there was some degree of consensus among the raters, variations in interpretation existed, likely stemming from differences in individual strategies when assessing the Perde^{Çeşni} changes. However, additional analysis uncovered a substantial level of agreement among the expert raters when evaluating the Perde^{Çeşni} changes with the lowest five and highest five means, yielding a Krippendorff's Alpha value of 0.720. This finding supports the notion of a "gray area" regarding the transition cost of some Perde^{Çeşni} changes, influenced by personal strategies derived from the experiences of musicians.

Additionally, we evaluated our algorithm's performance by comparing it with expert ratings using four different distance functions: Cosine, Minkowski, Manhattan, and Euclidean distances can be observed from Figure 4. Spearman correlation tests revealed significant and positive correlations between our algorithm and the expert results, Cosine distance giving the highest correlation of 0.828 with p <.001 (Table 1).

These results demonstrate strong and statistically significant correlations between our algorithm's predictions and expert judgments, across various distance functions, indicating the effectiveness of our approach in predicting the perceived difficulty of executing the Perde^{Çeşni} changes.

		Expert Rating
Cosine	Spearman's rho	0.828
	p-value	<.001
Minkowski	Spearman's rho	0.695
	p-value	<.001
Manhattan	Spearman's rho	0.659
	p-value	<.001
Euclidean	Spearman's rho	0.662
	p-value	<.001

Table 1. Spearman Correlation Tests: Experts' Evaluations versus Distance Functions.

6. CONCLUSION

This paper contributes to the understanding and quantification of distance and similarity among perde and çeşni components within Turkish Makam Music. Our investigation revealed the inadequacy of existing models in capturing the nuanced differences in distance within Makam music, particularly concerning modal shifts. By drawing from established music theory principles and cognitive approaches to distance measurement, we developed a novel approach rooted in geometric representations, specifically the Tonnetz model. Through the construction of this framework, we aimed to address the challenge of accurately representing the varying significance of relationships between different melodic progressions.

Octave equivalence poses a unique challenge in Turkish music, as perde names vary according to registral differences (e.g., *Yegâh-Nevâ-Tiz Nevâ, Rast-Gerdaniye, Dügâh-Muhayyer*). This variability complicates the application of standard tonnetz frameworks, which typically rely on fifths and thirds. To address this, we extended the model to incorporate octave relations, thus devising a Makam Perde Space (PMAKAM) represented as a 3-D Tonnetz. This enhancement allows us to elucidate the distance between Perde^{Çeşni} structures in terms of thirds, fifths, and octaves, providing a more accurate depiction of Makam music's structural relationships.

Our algorithmic approach facilitated the calculation of distances between different *Çeşni* structures, particularly those sharing primary axis relations. By comparing these results with expert ratings, we demonstrated the efficacy of our algorithm in predicting the perceived difficulty of executing Perde^{Çeşni} transformations.

However, our study is not without limitations. Notably, we treated "perde" as fixed points in space, neglecting their movable quality within a frequency bandwidth. This oversimplification may limit the applicability of our findings in contexts where "perde" exhibit greater flexibility. Additionally, our focus was primarily on a specific subset of Perde^{Çeşni} transformations within Turkish Makam Music, particularly those based on the same perde axis (fixed-axis). Future research should broaden this scope to encompass a wider range of transformations, including intra-connected and transference relations. Lastly, it is important to note that Makam music encompasses a rich array of expressive elements beyond pitch, including rhythm, ornamentation, and timbre, which were not fully addressed in our study. All these factors might possibly contribute to subjective variability in musical evaluation leading to the moderate level of agreement among expert raters in the "gray area." Future research could integrate these aspects to create a more holistic representation of Makam music, allowing for a deeper understanding of its expressive nuances.

By combining theoretical insights with computational approaches, we pave the way for a more comprehensive understanding of the intricate relationships within Turkısh Makam Music, thereby enriching both theoretical discourse and practical applications in musicology and computational music analysis.

Acknowledgments

This work is supported by the Scientific and Technological Research Council of Turkey, TÜBİTAK, Grant 122K923.

7. REFERENCES

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