TRAJECTORY DESCRIPTORS: AN APPROACH TO MUSIC GENRE CLASSIFICATION THROUGH CURVES AND VECTORS IN THE TONNETZ

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ABSTRACT

We present an approach to geometrically represent and analyze the harmonic content of musical compositions based on a formalization of chord sequences as spatial trajectories. This allows us in particular to introduce a toolbox of novel descriptors for automatic music genre classification. Our analysis method first of all implies the definition of harmonic trajectories as curves in a type of geometric pitch class spaces called Tonnetz. We define such curves by representing successive chords appearing in chord progressions as points in the Tonnetz and by connecting consecutive points by geodesic segments. Following a recently established hypothesis that assumes the existence of a narrow link between the musical genre of a work and specific geometric properties of its spatial representation, we introduce a toolbox of descriptors relating to various geometric aspects of the harmonic trajectories. We then assess the appropriateness of these descriptors as a classification tool that we test on compositions belonging to different musical genres. In a further step, we define a representation of transitions between two consecutive chords appearing in a harmonic progression by vectors in the Tonnetz. This allows us to introduce an additional classification method based on this vectorial representation of chord transitions.

1. INTRODUCTION

Using the Tonnetz – that is primarily defined to describe a spatial organization of pitch classes – as the main framework for automatic music genre or music style classification is a task put forth by recent works in Computational Musicology and Music Information Retrieval.

In terms of common formalization and usage, the Tonnetz can be considered to be a two-dimensional triangular grid, whose vertices represent different pitch classes $p \in \mathbb{Z}/12\mathbb{Z}$. The neighboring relationships between these pitch classes are determined by three pitch-class intervals $i_k \in \mathbb{Z}/12\mathbb{Z}$ (for $k \in \{1, 2, 3\}$), that describe the intervallic relations between pitch classes along each of the three axes

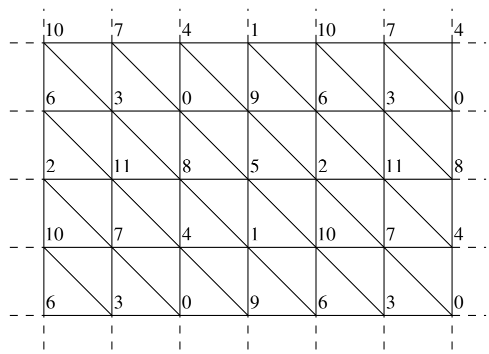

Figure 1. Representation of the Tonnetz based on a two-dimensional triangular grid. The different vertices of the grid represent pitch classes from 0 (or C) to 11 (or B). In this example, the neighboring relationships between the different pitch classes are generated by the pitch class intervals $i_1 = 3$ (along the horizontal axes), $i_2 = 4$ (along the vertical axes) and $i_3 = 5$ (along the oblique axes).

of the Tonnetz grid. An example of a geometric realization of the Tonnetz is illustrated in Figure 1.

In recent approaches, Bigo et al. [1,2] and Karystinaios et al. [3] introduce formalizations that allow to represent the entire harmonic content of musical pieces by trajectories in the Tonnetz. More precisely, by basing themselves on a vision of the Tonnetz as a simplicial complex, they propose to represent chord progressions by *chord complexes*. These chord complexes are constructed as the union of the pitch classes forming the considered chords and all the edges connecting these pitch classes. Louis Bigo then establishes the hypothesis that there exists a significant link between the style assigned to musical – and especially harmonic – content and certain geometric aspects of the spatial representation of this musical content. In particular, he introduces a measure to assess the compactness of such a representation in the Tonnetz.

Following the research paradigm implying such harmonic trajectories, we present, in this paper, a novel method to spatially represent chord progressions by curves in the Tonnetz. Contrary to the works of Bigo et al. and Karystinaios et al., our mathematical formalism involves describing the Tonnetz as a continuous space by embedding it on a flat torus (section 2.1). This allows us to define a procedure to represent chords by individual points in this continuous Tonnetz (section 2.2). Relying on this theoretical context, we then construct curves – our Tonnetz *trajectories* – that represent entire chord progressions by gradually connecting the positions in the Tonnetz of successive chords (section 2.3). Once these trajectories have been constructed, we consider a formalism that allows us to represent transitions between two consecutive chords as vectors in the Tonnetz, that we shall call *transition vectors* (section 2.4).

In order to measure various geometric aspects of our Tonnetz trajectories, we introduce a toolbox of several *trajectory descriptors* that are conceived to relate to specific geometric properties and spatial qualities of the trajectories (section 3). We then use both *trajectory descriptors* and *transition vectors* as features for automatic style classification (section 4).

Through our research, we explore a novel way of geometrically representing and analyzing musical harmony. By defining Tonnetz *trajectories*, *transition vectors* and *trajectory descriptors*, we propose novel formalisms and tools developed for analytical tasks in computational music analysis and music theory. By using our tools as classifier features, we demonstrate novel applications in the field of automatic style classification of music.

2. TONNETZ TRAJECTORIES

2.1 Embedding the Tonnetz on a flat torus

The geometric and topological properties of the Tonnetz and its generalizations as well as its suitability as a tool for harmonic analysis have been the subject of numerous mathematical and musicological studies (see e.g. [4–9]). For the purposes of this article, we restrict our approach to the Tonnetz generated by the pitch class intervals 3, 4 and 5 that we initially consider to be a pitch class based, labeled graph. In this graph, the different nodes are labeled by the 12 pitch classes such that each pitch class is represented exactly once and such that the neighbors of each pitch class are exactly the pitch classes that are either a minor third, a major third or a fifth away.

Our actual formalization of the Tonnetz – that, henceforth, we shall denote by $\mathcal{T}(3,4,5)$ – is then based on the observation that this graph is toroidal (see e.g. [5] for an in-depth study of the geometric and topological properties of different Tonnetz generalizations). This shall allow us to naturally embed the graph of the Tonnetz on a flat torus [1]. To be more precise, we propose to consider the rectangular flat torus [2]

$$\mathbb{T} := \mathbb{R}^2 \big/ (4\mathbb{Z} \times 3\mathbb{Z}), \tag{1}$$

together with the canonical projection

$$\pi : \mathbb{R}^2 \to \mathbb{T}. \tag{2}$$

We then define our embedding of the Tonnetz graph on this specific rectangular flat torus by associating, for $i, j \in \mathbb{Z}$, the node labeled by the pitch class $(-3i + 4j) \bmod 12$ with the point $\pi(i, j)$ on the flat torus.

[1] (see e.g. [10, 11] for a detailed theoretical framework on flat tori)

[2] An equivalent definition describes this rectangular flat torus as the quotient space of $\mathbb{R}^2$ under the identifications $(x, y) \sim (x + 4, y) \sim (x, y + 3)$ for all $(x, y) \in \mathbb{R}^2$.

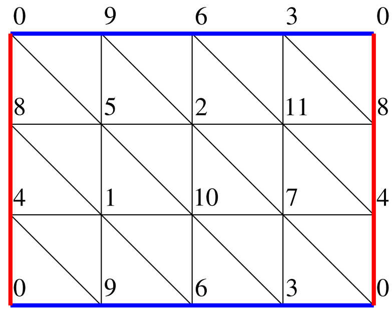

Figure 2. Visualization of the Tonnetz space $\mathcal{T}(3,4,5)$. The embedding of the Tonnetz on a flat torus is characterized by the rectangular *fundamental region* of the flat torus with a pairwise identification of the opposites sides.

As visualized in Fig. 2, the resulting Tonnetz space corresponds to the rectangular *fundamental region* of the underlying flat torus that pairwise identifies its opposites sides. The pitch class 0 (or C) becomes assigned to the point with coordinates $(0, 0)$. All horizontal axes represent interval leaps corresponding to the pitch class interval 3, vertical axes correspond to the pitch class interval 4, oblique axes to the pitch class interval 5.

2.2 Placing chords in the Tonnetz

Aiming to use the Tonnetz as a harmonic space, we follow the examples of Bigo et al. [1, 2] and Karystinaios et al. [3] and propose to expand the representation potential of the Tonnetz as a pure *pitch class space* to that of a *pitch class set space*. The two approaches mentioned tackle this representation problem by considering chords as being represented in the Tonnetz by chord complexes built up from all the pitch classes defining the chord in question as well as the edges connecting these pitch classes.

In contrast to this approach, we propose a method that represents chords as isolated points (rather than larger complexes) in the Tonnetz. Therefore, we suggest to determine the centroid of all the pitch classes defining a chord.

More precisely, given a chord in the form of a pitch class set $\mathbf{C}$ of size $N \in \{1, \ldots, 12\}$, we consider the set of all nodes $\{P_n\}_{n \in \{0, \ldots, N-1\}}$ in the Tonnetz space $\mathcal{T}(3,4,5)$ (as defined in section 2.1 via an embedding on a flat torus) that are labeled by the pitch classes defining $\mathbf{C}$.

In a second step, we *unfold* our flat torus Tonnetz in order to obtain, for $n \in \{0, \ldots, N-1\}$, the preimages $\pi^{-1}(\{P_n\}) \subset \mathbb{R}^2$ under the projection map π of these labeled nodes.

Among these preimages, we then construct all the N-polygons [3] $\mathcal{P}$ of minimum perimeter whose corners are labeled exactly once by the different pitch classes of the initial pitch class set $\mathbf{C}$.

In a last step, we determine the centroids (or geometric centers) $G_{\mathcal{P}}$ of all these polygons of minimum perimeter and place them back into Tonnetz space $\mathcal{T}(3,4,5)$ via

[3] (i.e. polygons with exactly N corners)

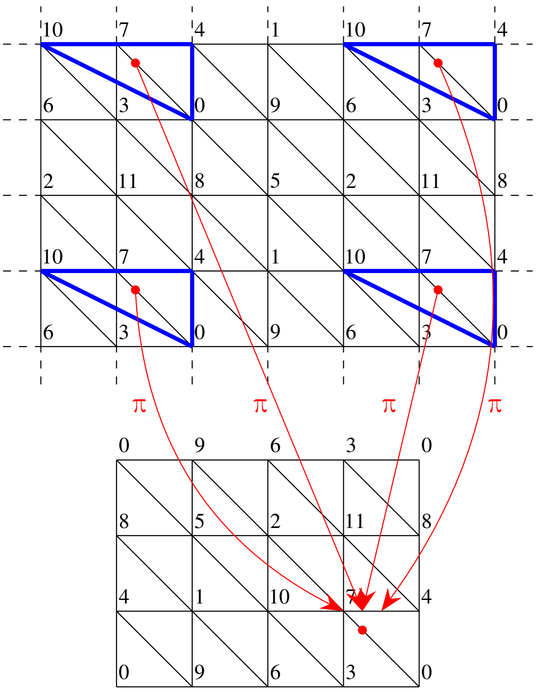

Figure 3. Illustration of the placement procedure for the pitch class set $[0, 4, 7, 10]$ (corresponding to the chord C^7) in the Tonnetz $\mathcal{T}(3, 4, 5)$. We therefore place ourselves in an *unfolded* version of the Tonnetz *(upper part of the figure)*. In this unfolded Tonnetz embedded in $\mathbb{R}^2$, we first determine all the polygons bounded by the pitch classes 0, 4, 7 and 10 that have minimum perimeter *(marked in blue)* and computer their centroids *(marked in red)*. These centroids are then transported back to the original *folded* version of the Tonnetz via the projection map π and finally define the position that represents our given pitch class set $[0, 4, 7, 10]$ in the Tonnetz.

the projection map π.[4] Finally, we choose to define the placement of the pitch class set $\mathbf{C}$ by the set of all points $\pi(G_{\mathcal{P}}) \in \mathcal{T}(3, 4, 5)$ found through this procedure.
In Fig. 3, we illustrate the hereabove described placement procedure for the pitch class set $[0, 4, 7, 10]$, corresponding to the chord C^7.

Having defined a procedure to represent chords (given in the form of pitch class sets) by isolated points in the Tonnetz, we are now set to represent chord progressions[5] by *trajectories* (defined in the following section 2.3) and chord transitions by *vectors* (defined in section 2.4) in our Tonnetz space $\mathcal{T}(3, 4, 5)$.

[4] In this step, it becomes apparent why it is necessary to first *unfold* the Tonnetz space. Indeed, some chords, such as the chord described by the pitch class set $[1, 4, 7, 10]$, allow multiple equally valid positions in the Tonnetz. By unfolding the Tonnetz, line segments connecting the pitch class 4 to the pitch class 7, or connecting the pitch class 1 to the pitch class 4, or connecting the pitch class 10 to the pitch class 1, or finally, connecting the pitch class 7 to the pitch class 10, all describe polygons of minimum perimeter that pass through all the pitch classes of the chord. Thus, these line segments induce four valid positions for the pitch class set $[1, 4, 7, 10]$ in the Tonnetz under the projection map π.

[5] In the following sections, we shall use the term *chord progression* to denote a succession of pitch class sets, the term *chord transition* to denote a chord progression of length 2, i.e. a succession of two directly consecutive pitch class sets.

2.3 Trajectories representing chord progressions

Basing ourselves on the above described procedure to represent chords by points in the Tonnetz, we propose to introduce a method to represent chord progressions by curves. Intuitively, given a chord progression $(\mathbf{C}_0, \ldots, \mathbf{C}_{N-1})$, we therefore trace a piecewise affine curve in the Tonnetz that connects the successive positions associated with the different pitch class sets $\mathbf{C}_n$ (for $n \in \{0, \ldots, N-1\}$).

As a matter of comprehensibility, we here present our construction algorithm for such Tonnetz trajectories by assuming that the first chord of a given chord progression admits only one unique representation, i.e. as a single point in the Tonnetz.[6]

Under this assumption, the construction algorithm for Tonnetz trajectories is iteratively established as follows:

First of all, the position of the initial chord $\mathbf{C}_0$ shall define the starting point $x_0 \in \mathcal{T}(3, 4, 5)$ of the trajectory.

Then, for $n = 1, \ldots, N-1$, it is necessary to distinguish between two cases.

If the chord $\mathbf{C}_n$ is represented by a unique point x_n in the Tonnetz, we newly construct the minimal geodesic segment joining the position $x_{n-1} \in \mathcal{T}(3, 4, 5)$ (which is the position of the previously placed chord $\mathbf{C}_{n-1}$) to the point $x_n \in \mathcal{T}(3, 4, 5)$.

If, however, the chord $\mathbf{C}_n$ admits several positions in the Tonnetz, we propose to arbitrarily privilege the position closest to the point $x_{n-1} \in \mathcal{T}(3, 4, 5)$ (which is, again, the position of the previously placed chord $\mathbf{C}_{n-1}$). If then, still more than one point comes into question, we privilege the position such that the oriented angle between the vector $(1, 0)$ and the velocity vector of the newly constructed geodesic segment becomes minimal.

In any of these cases, we denote the newly constructed minimal geodesic segment by $\gamma_n : [n-1, n] \to \mathcal{T}(3, 4, 5)$.

Our final trajectory shall eventually be given by the concatenation of the $N-1$ found geodesic segments γ_n (for $n = 1, \ldots, N-1$).

By way of example, we show, in Fig. 4, the step by step construction of the trajectory representing the chord progression Cmaj – C^7 – Fmaj – F♯dim^7 – or, in terms of pitch class sets, the progression $([0, 4, 7], [0, 4, 7, 10], [0, 5, 9], [0, 3, 6, 10])$.

2.4 Transition vectors representing chord transitions

Based on this trajectory construction procedure, we naturally define a way of representing a transition between two chords by a vector in the Tonnetz. For this purpose, let $(\mathbf{C}_0, \mathbf{C}_1)$ be a chord transition represented by a trajectory consisting of a geodesic segment between two points x_0 and x_1 in the Tonnetz space $\mathcal{T}(3, 4, 5)$. We furthermore consider the parametrization:

$$\gamma : [0, 1] \to \mathcal{T}(3, 4, 5), \tag{3}$$

[6] As seen in section 2.2, some chords, especially chords showing specific symmetries, indeed admit more than one valid position in the Tonnetz. Diminished seventh chords, for instance, are represented by 4 symmetrically distributed points.

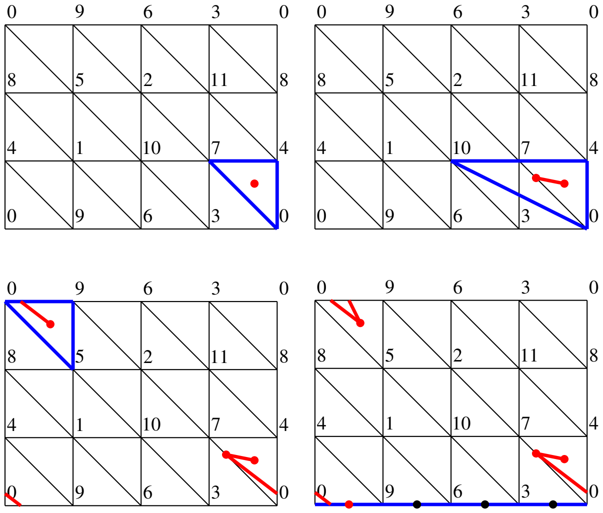

Figure 4. Step by step construction of the trajectory representing the chord progression $([0, 4, 7], [0, 4, 7, 10], [0, 5, 9], [0, 3, 6, 10])$ The beginning of the trajectory is given by the minimal geodesic segment connecting the positions of the chords $[0, 4, 7]$ and $[0, 4, 7, 10]$ *(upper right part of the figure)*. We then determine the minimal geodesic segment between the positions of the chords $[0, 4, 7, 10]$ and $[0, 5, 9]$, that visually passes through the four sides of the rectangular fundamental region of the Tonnetz space *(lower left part of the figure)*. The last step requires determining the point closest to the chord $[0, 5, 9]$ among the four points representing the chord $[0, 3, 6, 9]$ in the Tonnetz *(lower right part of the figure)*.

with $\gamma(0) = x_0$ and $\gamma(1) = x_1$ of this geodesic segment. Then, we define the *transition vector* associated with the chord transition $(\mathbf{C}_0, \mathbf{C}_1)$ by the velocity vector:

$$\gamma'(0) \in T_{x_0}\mathcal{T}(3, 4, 5) \cong \mathbb{R}^2, \tag{4}$$

where $T_{x_0}\mathcal{T}(3, 4, 5)$ denotes the tangent space of $\mathcal{T}(3, 4, 5)$ in the point x_0.

3. TRAJECTORY DESCRIPTORS

As an essential part of the framework of our research, we globally follow the paradigm – significantly put forth by the works of Bigo et al. [1] and by Karystinaios et al. [3] – that presumes the existence of a narrow link between the stylistic nature assigned to the harmonic content of a musical piece and specific geometric characteristics of its corresponding spatial representation. In order to measure or capture various properties of our Tonnetz trajectories, we introduce a toolbox of several descriptors that we specifically designed to describe geometric aspects of these trajectories and that can potentially be related to harmonic characteristics of the represented chord progressions.

In Table 1, we give an overview of all the trajectory descriptors introduced below.

Later, in section 4, we propose an application towards testing these trajectory descriptors as features implied in an automatic classification tool for chord progressions.

In the following sections, we assume that a Tonnetz trajectory of length N, i.e. a trajectory representing a chord progression of length N, is given by the parametrization:

$$\Gamma : \; [0, \; N-1] \; \rightarrow \; \mathcal{T}(3, 4, 5), \tag{5}$$

where, for $n = 0, \ldots, N-1$, the point $\Gamma(n)$ corresponds to the position associated to the trajectory's n^{th} chord in the Tonnetz.

3.1 The number of different transition vectors

From the definition of transition vectors introduced in section 2.4, it follows intuitively that chord transitions of the same nature are represented by vectors of the same length and direction. [7] In particular, this entails that any succession of specific scale degrees (e.g. the progression $\mathbf{II} - \mathbf{V^7} - \mathbf{vi}$ in a major scale) is represented by the same succession of transition vectors when being transposed.

This simple observation leads us to introduce a first trajectory descriptor that counts the number of different transition vectors that appear within a given trajectory. This shall allow us to obtain a first estimation on the variety of chord transitions used in a chord progression and, thereby, an estimation on the richness of the harmonic language in use.

3.2 The number of different vector orientations

As a descriptor that is closely linked to our first descriptor (counting the number of different transition vectors in a trajectory), we propose to additionally take count of the number of different vector orientations. Practically, we therefore consider collinear vectors as being part of a same *collinearity class* and by counting the number of *collinearity classes*. Geometrically, this boils down to counting the number of axes in the Tonnetz along which a given trajectory evolves.

3.3 The number of triangular regions visited by a trajectory

The Tonnetz $\mathcal{T}(3, 4, 5)$ being generated by interval classes corresponding to the minor and major thirds as well as the fifth, it naturally turns out to be a geometric space that is well-suited for representing major and minor triads. Indeed, it is obviously apparent that the pitch classes defining major and minor chords delimit triangular regions inside the Tonnetz space $\mathcal{T}(3, 4, 5)$. In order to estimate the tonal mobility of a given chord progression, we propose to take advantage of this property and count the number among these triangular regions that the trajectory of this chord progression passes through. We particularly reckon that highly modulation progressions, by visiting a high number of tonalities, imply trajectories that traverse a high number of triangle regions.

[7] The formal framework presented in this paper does not, however, allow to establish a one-to-one correspondence between the set of possible transition vectors and the set of all possible chord transition natures. In other terms, two vectors of the same length and direction may represent two chord transitions of different nature. A more refined representation method, for instance by assigning different heights to different chord types, may help remedy this issue, but is not required in the context of the basic definitions of our trajectory descriptors.

3.4 Distances between chords and angles between transition vectors

Still pursuing the idea of estimating characteristic aspects of the tonal path of a composition, we take into account the average distance between two successive chords. More formally, given the parametrization of a Tonnetz trajectory as defined by (5), the average distance between two successive chords can be given by the formula:

$$\overline{\text{distances}}(\Gamma) := \frac{1}{N}\sum_{n=0}^{N-1} d_{\mathcal{T}(3,4,5)}\Big(\Gamma(n),\, \Gamma(n{+}1)\Big), \quad (6)$$

where $d_{\mathcal{T}(3,4,5)}$ denotes a distance function on the Tonnetz space $\mathcal{T}(3,4,5)$.

As a further descriptor that estimates the average evolution of a trajectory, we propose to also assess the angles between consecutive transition vectors. Formally, we therefore determine the absolute values of the cosine similarity between two consecutive transition vectors. [8] Our descriptor is then given by the average of these values along an entire trajectory i.e., considering again the parametrization (5), we obtain:

$$\overline{\text{cos similarities}}(\Gamma) := \frac{1}{N-1}\sum_{n=0}^{N-2}\left|\frac{\mathbf{v}_n \cdot \mathbf{v}_{n+1}}{\|\mathbf{v}_n\|\,\|\mathbf{v}_{n+1}\|}\right|, \quad (7)$$

where, for $n = 0, \ldots, N-2$, $\mathbf{v}_n$ and $\mathbf{v}_{n+1}$ denote the transition vectors originating from the n^{th} and the $(n+1)^{\text{th}}$ chord respectively and where $\mathbf{v}_n \cdot \mathbf{v}_{n+1}$ denotes the standard Euclidean dot product between these two vectors.

Having introduced these two descriptors, we additionally propose to include in our descriptor toolbox the standard deviation associated with these latter average values.

3.5 The width of a trajectory

In order the get a certain impression of the *width* of a trajectory (starting with chord $\Gamma(0)$ and ending with chord $\Gamma(n-1)$ according to the parametrization (5)), we propose to compare it to a straight line connecting $\Gamma(0)$ with $\Gamma(n-1)$. As such a comparison turns out to be not trivial in a quotient space such as our flat torus Tonnetz space, we propose again to *unfold* our Tonnetz and, thereby, also unfold our given trajectory.

Similar to the unfolding done in section 2.2, we therefore consider the preimage under our canonical projection π of the given trajectory $\Gamma([0,\, N-1])$. The obtained preimage $\pi^{-1}\Big(\Gamma([0,\, N{-}1])\Big)$ shall then correspond to unfolded and connected versions of the initial trajectory that are now embedded in $\mathbb{R}^2$. Among these preimage trajectories embedded in $\mathbb{R}^2$, it shall be sufficient for our purposes to select, as a representative, an arbitrary trajectory that we denote by $\widetilde{\Gamma}$. Through this unfolded version of our trajectory, we finally, assess its width by computing the Hausdorff distance between the curve $\widetilde{\Gamma}([0,\, N-1])$ and the straight line connecting its starting and end points $\widetilde{\Gamma}(0)$ and $\widetilde{\Gamma}(N-1)$.

More precisely, we denote by σ the straight line passing through $\widetilde{\Gamma}(0)$ and $\widetilde{\Gamma}(N-1)$ in $\mathbb{R}^2$ and by Σ^+ and Σ^- the half-planes of $\mathbb{R}^2$ determined by σ. Then, our width descriptor shall be defined by the formula:

$$\text{width}(\Gamma) := d_{\text{H}}(\widetilde{\Gamma} \cap \Sigma^+,\, \sigma) \;+\; d_{\text{H}}(\widetilde{\Gamma} \cap \Sigma^-,\, \sigma), \quad (8)$$

where d_{H} denotes the Hausdorff distance in $\mathbb{R}^2$.

3.6 The inconstancy of a trajectory

Our final descriptor is largely inspired by the work of Jean-Paul Allouche and Laurence Maillard-Teyssier [12] on the *inconstancy* of sequences and curves. The idea that we adopt in the context of our paper is to assess the global aspect of a given curve in $\mathbb{R}^2$ in terms of its fluctuations or its curviness, i.e. the more a curve looks irregular, the higher its *inconstancy* should be, whereas the more it resembles to a straight line, the lower its *inconstancy* should be (and eventually tend to 1).

As the inconstancy formula presented by Jean-Paul Allouche and Laurence Maillard-Teyssier implies curves embedded in the Euclidean plane, we shall again, as described in the previous section , unfold any given Tonnetz trajectory Γ in order to obtain a corresponding unfolded trajectory $\widetilde{\Gamma}$ embedded in $\mathbb{R}^2$.

We then can express, according to the theoretical framework described in [12], the inconstancy of the Tonnetz trajectory Γ by the formula:

$$\text{inconstancy}(\Gamma) := \frac{2 \cdot \text{len}(\widetilde{\Gamma})}{\text{per}\big(\text{conv}(\widetilde{\Gamma})\big)}, \quad (9)$$

where $\text{len}(\widetilde{\Gamma})$ denotes the length of the curve $\widetilde{\Gamma}$ and $\text{per}\big(\text{conv}(\widetilde{\Gamma})\big)$ the perimeter of its convex hull in $\mathbb{R}^2$.
We introduce this descriptor particularly to verify its adequateness to keep track of the potential tonal unsteadiness and harmonic richness of given chord sequences. As illustrated by Fig. 5, our inconstancy descriptor specifically returns the value 1 for a sequence of descending fifths, while it returns a higher value for a sequence that moves more irregularly through different chords and/or tonalities.

4. APPLICATIONS TO STYLE CLASSIFICATION

In order to assess the suitability of our trajectory formalism and specifically the trajectory descriptors introduced in section 3 as a tool for automatic style classification, we propose two applications – using, first of all, trajectory descriptors and, in a second step, transition vectors as features for classification.

4.1 Data sets, chord extraction and preprocessing

Our method implies the use of MIDI file data sets that provide pieces belonging to a range of different musical styles from which we retrieve the harmonic content in order to compute their corresponding trajectories in the Tonnetz. For all analyses performed within the context of this paper, we use MIDI files belonging to the MuseData corpus [13] and the Nottingham database [14]. We collected

[8] This shall give us values closer to 0 the closer two vectors are to being orthogonal, values closer to 1 the closer two vectors are to being collinear.

Descriptor name	*Explanation*
$\lvert\{\text{transition vectors}\}\rvert$	Counts the number of different transition vectors that appear within a given trajectory.
$\lvert\{\text{vector orientations}\}\rvert$	Considers collinear vectors as being part of a same *collinearity class* and counts the number of different *collinearity classes* within a trajectory.
$\lvert\{\text{visited triangular regions}\}\rvert$	Counts the number of triangular regions of the Tonnetz grid that a trajectory passes through.
$\overline{\text{distances}}$	Computes the average distance between two successive chords.
$\text{std}(\text{distances})$	Computes the standard deviation of all the distances between two successive chords.
$\overline{\text{cos similarities}}$	Computes the average cosine similarity between two successive transition vectors.
$\text{std}(\text{cos similarities})$	Computes the standard deviation of all the cosine similarities between two successive transition vectors.
width	Computes the width of a trajectory by considering the Hausdorff distance between the *unfolded* version of the trajectory and the straight line connecting its starting and end points.
inconstancy	Estimates the global aspect of a trajectory in terms of its fluctuations or its unsteadiness.

Table 1. Overview of all the trajectory descriptors defined in section 3.

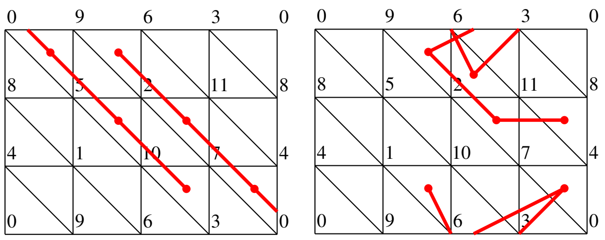

Figure 5. Comparison between the trajectories of the chord progression Dmaj – Gmaj – Cmaj – Fmaj – B♭maj – E♭maj – correspoding to a sequence of descending fifths – *(left side of the figure)* and the chord progression F♯maj – Bmin – Cmaj – Dmaj – Gmaj – Emaj *(right side of the figure)* in terms of their inconstancies. While the first, very regular progression has an inconstancy of 1.0, the second progression, seeming more irregularly fluctuating, has an inconstancy of about 2.17.

284 pieces in total from the MuseData corpus (including chorale harmonizations by J. S. Bach, Trio Sonatas by A. Corelli and various pieces by W. A. Mozart) and 287 pieces from the Nottingham database (consisting of a large collection of British and American folk tunes). Throughout the following sections, these four stylistic contexts shall define our target classes (referred to as *Corelli*, *Bach*, *Mozart* and *Nottingham*) for all classification tasks undertaken.

For each of the MIDI files in our data set, we extract the harmonic content in the form of sequences of pitch class sets using the Python library *music21* [15]. Then, for each of theses sequences of pitch class sets (or, returning to the term used above in this paper, for each chord progression), we determine its Tonnetz trajectory. Subsequently, we consider all the values measured using our trajectory descriptors as well as all the transition vectors implied by the chord transitions represented by the trajectory.

4.2 Trajectory descriptor based classification

In order to implement a trajectory descriptor based classification method, we propose to base ourselves on the k-nearest neighbors (k-NN) algorithm that we use in the style of binary and multiclass classifiers.

Therefore, we first perform measures using our descriptors on the trajectories of randomly chosen pieces belonging to two or more different styles represented in our database. As we are dealing with descriptors that might be impacted by the lengths of the analyzed trajectories, we propose, at this stage of our method, to randomly select trajectory extracts of the same length N to eliminate any length-related bias. More precisely, each of the pieces of our database shall be represented by exactly one trajectory of length N that is selected from a random starting position within the considered piece. Throughout all the classification tests described here below, we propose to set the length of each of the computed trajectories to $N = 20$.

In a next step, we apply the different descriptors defined in section 3 to each of these extracted trajectories and normalize the obtained values with respect to their mean and standard deviation.

In a final step, we use the k-NN algorithm to assess the adequateness of our trajectory descriptors as features for binary and multiclass classification. For this purpose, we choose, for each series of tests, two to four classes present in our data set (e.g. Bach chorales and folk tunes from the Nottingham database for a series of binary classification tests). We then split the chosen data into a training and a test set such that the test set contains exactly 10 pieces (represented by their respective trajectory extracts). We eventually train our k-NN classifier on the training set and evaluate its predictions with respect to the test set using the *accuracy* metric implemented by the formula:

$$\text{accuracy} := \frac{\text{number of correctly classified pieces}}{\text{total number of pieces}}. \quad (10)$$

In Table 2 and Table 3, we give an overview of all the re-

Styles to classify		*Avg. accuracy*
Corelli	Bach	0.80
Corelli	Mozart	0.82
Corelli	Nottingham	0.95
Bach	Mozart	0.84
Bach	Nottingham	0.94
Mozart	Nottingham	0.92

Table 2. Results obtained by using trajectory descriptors as features for binary classification.

Styles to classify	*Avg. accuracy*
Corelli, Bach, Mozart	0.69
Corelli, Bach, Nottingham	0.73
Corelli, Mozart, Nottingham	0.73
Bach, Mozart, Nottingham	0.75
Corelli, Bach, Mozart, Nottingham	0.67

Table 3. Results obtained by using trajectory descriptors as features for multiclass classification.

sults obtained throughout our descriptor based binary and multiclass classification tests.

All the tests in the series referred to in Table 2 and Table 3 were performed by using values obtained using six of our total of nine trajectory descriptors: the *number of different transition vectors*, the *number of different vector orientations*, the *number of visited triangular regions*, the *average distance between successive chords*, the *standard deviation of all these distances* and the *inconstancy*. We justify this selection by our observation that this specific set of descriptors allowed us to achieve the most consistent results over all our test series.

For each of the considered classes, we repeated our testing procedure 10 times by randomly resplitting the respective data sets each time. We provide, in Table 2 and Table 3, the average value of all the accuracy scores obtained across each of these series of 10 tests.

4.3 Transition vector based classification

As a second classification method explored during our research, we propose to rely exclusively on the nature of the different transition vectors retrieved from the harmonic content in our data sets. Thereby, instead of relying on global characteristics of geometric representations as measured by our trajectory descriptors, we focus, in this section, on local aspects of Tonnetz trajectories as expressed by individual transition vectors. From a music theoretical point of view, we base this approach on the assumption that the harmonic style of a corpus becomes implicitly defined through its harmonic content and, specifically through its individual chord transitions. Moreover, we presuppose that two music corpora relating to two different harmonic styles can be distinguished through specific characteristic chord transitions that are contained in one corpus and that are not contained in the other corpus.

To reconnect with the formalism of our transition vectors, given two musical corpora containing pieces belonging to two distinguishable harmonic styles $\mathfrak{C}_1$ and $\mathfrak{C}_2$, we determine the Tonnetz trajectories of all the pieces contained in these two corpora and, subsequently, all the corresponding transition vectors. We denote the set of all transition vectors related to the corpus $\mathfrak{C}_1$ by $\mathfrak{V}_1$, the set of all transition vectors related to the corpus $\mathfrak{C}_2$ by $\mathfrak{V}_2$.

We then propose to introduce the set of transition vectors that are exclusively found in corpus $\mathfrak{C}_1$, and that do not occur at all in corpus $\mathfrak{C}_2$, that we denote by

$$\mathfrak{V}_{1\setminus 2} := \mathfrak{V}_1 \setminus \mathfrak{V}_2 \tag{11}$$

and, similarly, the set of transition vectors that are exclusively found in corpus $\mathfrak{C}_2$, and not present in corpus $\mathfrak{C}_1$:

$$\mathfrak{V}_{2\setminus 1} := \mathfrak{V}_2 \setminus \mathfrak{V}_1. \tag{12}$$

Our classification method can now be described as follows. Given an unknown [9] piece, we determine its Tonnetz trajectory and all of its corresponding transition vectors. If then, among these transition vectors, there are more vectors that can also be found in $\mathfrak{V}_{1\setminus 2}$, we assign the unknown piece to the harmonic style of corpus $\mathfrak{C}_1$. Otherwise, if there are more vectors that can also be found in $\mathfrak{V}_{2\setminus 1}$, we assign it to the harmonic style of corpus $\mathfrak{C}_2$.

In Table 4, we give an overview on several results obtained throughout our vector-related classification tests. The tests in this series were performed using different couples of data sets and by repeating the classification procedure 10 times using randomly selected pieces out of each data set and determining the average accuracy score obtained.

4.4 Comparison between descriptor based and vector based classification

Contrary to the use of trajectory descriptors as classifier features, the current design of our method for transition vector based classification is only optimized for binary classification. In order to compare both types of applications, we summarize the results obtained throughout all our binary classification test series in Table 2 (descriptor based classification) and Table 4 (vector based classification). Overall, the results obtained using transition vectors appear to be systematically more accurate, with only the test series involving the Nottingham class leading to comparable results.

Styles to classify		*Avg. accuracy*
Corelli	Bach	0.97
Corelli	Mozart	0.95
Corelli	Nottingham	1.00
Bach	Mozart	0.96
Bach	Nottingham	0.98
Mozart	Nottingham	0.96

Table 4. Results obtained by using transition vectors as features for binary classification.

[9] The term *unknown* would, in this context, signify that it is not known whether the piece would belong to corpus $\mathfrak{C}_1$ or corpus $\mathfrak{C}_2$.

On a more technical level, our vector based method requires a particularly careful compilation of the used data sets. In fact, the proposed procedure seems to be highly sensitive to the total amount of different transition vectors contained in the trajectories retrieved from the considered classes. We particularly notice that the use of imbalanced classes (in terms of numbers of different transition vectors) leads to considerably less accurate results.

5. CONCLUSIONS

We presented a formalization allowing to represent chord progressions by curves – the *trajectories* – and chord transitions by vectors – the *transition vectors* – in the Tonnetz. Based on this mathematical framework, we introduced a toolbox of novel *trajectory descriptors* relating to several specific geometric aspects of Tonnetz trajectories. We have shown that using trajectory descriptors and transition vectors have yielded promising results when being used as features in frameworks designed for automatic harmonic style classification. Therefore, future work should aim for a more comprehensive evaluation of the proposed descriptors, for example by using a larger number of styles. Moreover, we emphasize that, by definition, our Tonnetz trajectories can only account for a subset of all the aspects that could make up the richness of a harmonic language. Thus, future work may also imply a larger amount of harmonic parameters, such as chord positions, voice-leading, or harmonic rhythm, as well as other musical parameters in general, in particular rhythm and timbre. This may demand a more profound work on richer, higher-dimensional musical spaces and, eventually, a re-formalization of the concept of trajectories.

Specifically with regard to vectors, a further analysis might involve a reflection on a connection between the transition vectors presented in this paper with the theory of *harmonic vectors* by Nicolas Meeùs [16], which has already been applied in the context of stylistic analyses by Philippe Cathé [17].

While the testing results found throughout our research seem interesting, we also reckon that the formal framework on Tonnetz trajectories might have applications outside of a classification context. For instance, in an educational environment, tracing trajectories in the Tonnetz might become a useful learning or creative tool when implemented in a way that allows to draw curves by hand on a Tonnetz interface and, through this, to freely create new chord progressions.

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